

THE DYNAMIC ADJUSTMENT PATH FOR PERFECTLY FORESEEN CHANGES IN MONETARY POLICY*

Russell S. BOYER

University of Western Ontario, London, Ont., Canada N6A 3K7

Robert J. HODRICK

Carnegie-Mellon University, Pittsburgh, PA 15213, USA

This paper investigates the real effects of announced or perfectly foreseen changes in monetary policy produced by the Mundell–Tobin effect. A particularly good place to study these non-neutralities appears to be the period between announcement and implementation of new policies. At announcement the economy jumps to a new equilibrium path moving continuously even after implementation. All transition paths between announcement and implementation are established to be scalar multiples of each other. The path depends upon the structure of the economy and the discounted present value of the change in policy. Both algebraic and diagrammatical analyses are presented.

1. Introduction

Rational expectations models of fluctuations in real economic activity have concentrated upon unanticipated changes in the money supply as the important source of non-neutralities in monetary policy. In theory, real effects also can arise from announced or perfectly foreseen changes in the money supply. The mechanism which generates these non-neutralities is the Mundell–Tobin effect: namely, the effect of changes in the anticipated rate of inflation upon real interest rates on money and other assets which consequently affect the level of investment and eventually the capital stock.¹ Since the two types of non-neutralities have different effects on real output, it is important to control for both effects when designing empirical tests of the influence of monetary policy on real economic activity.

The major finding of this paper is that a particularly good place to study the Mundell–Tobin non-neutralities that arise in a rational expectations environment appears to be the period between the announcement and

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¹Mundell (1963) noted that an increase in the anticipated rate of inflation could lower the real interest rate in the short run. Tobin (1965) modeled the effects on the steady-state capital stock.

implementation of a new policy. The reasons for this are two-fold. First, under the rational expectations assumption, the dynamic path which the economy follows during this period is radically different from the path predicted by the older models employing static or adaptive expectations. At the time of the announcement, the economy jumps to a new equilibrium path thereafter moving continuously even after the implementation of a large policy change. Second, what has not been widely recognized is the extreme simplicity of the interim path. The reason is that aside from its scale, the interim path is independent of the change in policy. Thus, any one policy establishes a transition path which is a scalar multiple of any other policy change. In particular, the form of the transition path of the economy for an announced step increase in the future money supply is the same whether that increase is temporary or permanent. In this sense, the economy's dynamics during the transition period aside from the scale parameter are structural characteristics of the model.²

These findings are demonstrated here in a framework which is familiar to researchers in this field. Section 2 presents a continuous time analogue of the model employed by Fischer (1979). Although he considered only closed economy issues, his model has a natural open economy interpretation so that issues on exchange rate dynamics are discussed in this paper. Section 3 presents the general solution for the deviations from their former equilibrium path of the price level or the exchange rate and the capital stock for a shock to the future path of the money supply of general form. When the problem of revised expectations is viewed in terms of such deviations, the results of other researchers can be applied to a wider class of initial conditions than has previously been considered.

Section 4 presents the analysis of delays in implementation for a general shock to monetary policy that is announced some time before its implementation.

Sections 5 and 6 present these issues diagrammatically by examining a permanent and a temporary step increase in the money supply, and a change in its rate of growth. Finally, section 7 provides a conclusion.

2. The model

The model that is used in this paper is a continuous time analogue of the monetary growth model discussed in Fischer (1979).³ While the formal

²Hurwicz (1962) discusses the meaning of structural equation in an econometric context. Structural parameters are invariant to policy interventions. See footnote 12 for an elaboration.

³Fischer notes that his model can be derived from an overlapping generations model with maximizing behavior. While the long-run capital stock in Sidrauski's (1957) maximizing model is independent of the rate of monetary growth, Brock (1974) notes that including leisure in the utility function with consumption and real balances will introduce a non-neutrality if leisure and real balances are substitutable as seems likely.

analysis here will consider the variables in the closed economy framework employed by Fischer, we indicate throughout the paper how the model can be used to address monetary issues which have arisen in the literature on the dynamics of exchange rates in an open economy.⁴

In the closed economy version of the model, output is assumed to be produced by a fully-employed labor force of constant size in conjunction with a capital stock which is fixed at a moment in time but which can change in a continuous manner over time. Since the economy can also accumulate wealth in the form of real balances, the portfolio balance and savings and investment decisions describe the evolution of the economy over time. Such an economy can be represented by the following equations:

$$\dot{k} = \delta_0 - \delta_1 k - \delta_2(m - p) + \delta_3 \dot{p}, \quad (1)$$

and

$$m - p = \gamma_0 + \gamma_1 k - \gamma_2 \dot{p}, \quad (2)$$

where k is the natural logarithm of the capital stock, m is the logarithm of the nominal money stock which is assumed to be a function of time only, and p is the logarithm of the price level. A dot above a variable signifies its right-hand derivative with respect to time. All parameters in (1) and (2) are positive implying that the direction of influence of any variable can be determined from the sign that precedes it. Since it is assumed that foresight is perfect (except for the unanticipated changes in policy studied below), there is no need to distinguish between expected and actual rates of change for any variable.

The savings and investment behavior inherent in the goods market equilibrium is represented by (1). The rate of growth of the capital stock is postulated to depend negatively upon the existing capital stock and the stock of real balances and to depend positively upon the rate of inflation. The negative effects of the capital stock and real balances primarily reflect the influence of a long-run target for wealth. For the capital stock there is a second reason for the negative impact: an increase in the capital stock decreases the marginal product of capital which reduces the economy's long-run desired stock of capital. Similarly, an increase in the rate of inflation increases the growth rate of capital since economic agents desire to substitute out of real balances and into physical capital which is the Tobin effect.⁵

Eq. (2) is a conventional money demand function summarizing portfolio

⁴Kouri (1976), Flood (1979), and Boyer and Hodrick (1982) examine issues in the economics of the open economy in models which are formally similar to this one.

⁵We have modified Fischer's eq. (7) to allow for dependence of capital accumulation on wealth rather than just current income. Hence, $\delta_2 = 0$ in his model. In models in which the Tobin effect is not present, $\delta_3 = 0$ also.

balance between real balances and the capital stock. Demand for real balances depends positively on wealth or real income inducing the positive influence of the capital stock, and it depends negatively on the rate of inflation. The form of the money demand function guarantees that there can be no anticipated jumps in the price level since at the price discontinuity there would be an infinitely large expected rate of return to holding real balances. Prices can change discontinuously, but any such discontinuities must arise from unanticipated changes in policies.

An alternative, open economy interpretation of the model is possible. Consider a small open economy with flexible prices that takes foreign-currency prices and rates of return as given by the rest of the world. With this interpretation, p is the logarithm of the exchange rate; this is true since, by purchasing power parity, the logarithm of the domestic-currency price level is the sum of the logarithms of the exchange rate and the foreign-currency price level, and by choice of units, the latter price can be set to zero. With constant foreign prices, the domestic rate of inflation is equal to the time derivative of the exchange rate which is the appropriate argument in the money demand function. The variable k is interpreted as the level of net foreign assets with the specification of (1) justified by savings behavior alone. Since the rate of return is given by the world, changes in the marginal productivity of capital cannot be used to motivate the equation. Indeed, the accumulation of physical capital is ignored, but the fact that money is a non-traded asset in the small country model guarantees, through the wealth constraint, that k is given at a moment in time.⁶

This two equation model can be rewritten as

$$\dot{p} = \alpha_0 + \alpha_1(p - m) + \alpha_2 k, \quad (3)$$

and

$$\dot{k} = \beta_0 + \beta_1(p - m) - \beta_2 k, \quad (4)$$

where $\alpha_0 = \gamma_0/\gamma_2 = 0$ and $\beta_0 = (\delta_0 + \delta_3\gamma_0/\gamma_2) = 0$ by choice of units, $\alpha_1 = 1/\gamma_2 > 0$, $\alpha_2 = \gamma_1/\gamma_2 > 0$, $\beta_1 = (\delta_2 + \delta_3/\gamma_2) > 0$, and $\beta_2 = (\delta_1 - \delta_3\gamma_1/\gamma_2) > 0$ for δ_3 small.⁷ Eqs. (3) and (4) constitute a differential equation system in $p(t)$ and $k(t)$ which can be solved using standard techniques.⁸ The solution for the case in which $m(t)$ is the time-dependent forcing function is

⁶The small country models of Dornbusch (1976), Gray and Turnovsky (1979), and Wilson (1979) do not consider asset accumulation, but, instead, they motivate their dynamics with slow adjustment of prices in the domestic goods market.

⁷Fischer assumes that the ratio of the elasticities of the demand for capital with respect to the rate of return on capital and the rate of inflation are greater than the ratio of the same elasticities in money demand. This is equivalent to assuming that $\beta_2 > 0$.

⁸See Brauer and Nohel (1973) for a discussion of the solution technique.

$$\begin{aligned}
 p(t) = & C_2 q_2 e^{\lambda_2(t-T_0)} + \frac{q_1}{q_1 - q_2} \int_t^{\infty} \lambda_1 m(\tau) e^{-\lambda_1(\tau-t)} d\tau \\
 & + \frac{q_2}{q_1 - q_2} \int_{-\infty}^t \lambda_2 m(\tau) e^{-\lambda_2(\tau-t)} d\tau,
 \end{aligned} \tag{5}$$

and

$$\begin{aligned}
 k(t) = & C_2 e^{\lambda_2(t-T_0)} + \frac{1}{q_1 - q_2} \int_t^{\infty} \lambda_2 m(\tau) e^{-\lambda_2(\tau-t)} d\tau \\
 & + \frac{1}{q_1 - q_2} \int_{-\infty}^t \lambda_2 m(\tau) e^{-\lambda_2(\tau-t)} d\tau,
 \end{aligned} \tag{6}$$

where

$$\lambda_1 = [-(\beta_2 - \alpha_1) + \sqrt{(\beta_2 - \alpha_1)^2 + 4(\beta_1 \alpha_2 + \beta_2 \alpha_1)}] / 2 > 0,$$

$$\lambda_2 = [-(\beta_2 - \alpha_1) - \sqrt{(\beta_2 - \alpha_1)^2 + 4(\beta_1 \alpha_2 + \beta_2 \alpha_1)}] / 2 < 0,$$

$$q_1 = (\lambda_1 + \beta_2) / \beta_1 > 0 \quad \text{and} \quad q_2 = (\lambda_2 + \beta_2) / \beta_1 < 0.$$

T_0 is the time at which the capital stock is given by an initial condition. The limits of integration have been chosen such that the price level is determinate for $m(\tau)$ of exponential order less than λ_1 .⁹ The arbitrary coefficient multiplying the part of the homogeneous solution corresponding to the positive root, λ_1 , has been set to zero implying that the system cannot migrate indefinitely without being forced by the money supply process. Finally, the arbitrary coefficient, C_2 , must be chosen in order to satisfy the initial condition on the capital stock.

In the next section we consider the solution of the model for an arbitrary unanticipated change in the future time profile of the money supply while in section 4 we consider the impact of a delay in implementation.

3. Unanticipated changes in monetary policy

Suppose that, at some time in the evolution of the economy, agents become aware, perhaps through an announcement, that a new money supply time profile will occur in the future. Denote the new future path of the money supply by $\tilde{m}(\tau)$, and let the time of the announcement be T_0 to correspond to the notation in (5) and (6). The capital stock is given at the

⁹Such money supply processes are said to possess process consistency by Flood and Garber (1980). See Sargent and Wallace (1973) or Sargent (1979) for a discussion of the choices of the arbitrary constants and of the limits of integration.

time of the announcement, but the price level is free to jump since expectations of the future have changed. Agents are assumed to have perfect foresight of the future after the policy change, and they place zero probability on other paths.

To determine the new equilibrium price level process, $\tilde{p}(t)$, integrate the expression in (5) and (6) with the $\tilde{m}(\tau)$ process substituted for the $m(\tau)$ process. The linearity of the integration operator permits us to write these solutions in a more convenient fashion:

$$\begin{aligned} \tilde{p}(t) - p(t) \equiv p_d(t) = & C_{2d} q_2 e^{\lambda_2(t-T_0)} + \frac{q_1}{q_1 - q_2} \int_t^{\infty} \lambda_1 m_d(\tau) e^{-\lambda_1(\tau-t)} d\tau \\ & + \frac{q_2}{q_1 - q_2} \int_{-\infty}^t \lambda_2 m_d(\tau) e^{-\lambda_2(\tau-t)} d\tau, \end{aligned} \quad (7)$$

and

$$\begin{aligned} \tilde{k}(t) - k(t) \equiv k_d(t) = & C_{2d} e^{\lambda_2(t-T_0)} + \frac{1}{q_1 - q_2} \int_t^{\infty} \lambda_1 m_d(\tau) e^{-\lambda_1(\tau-t)} d\tau \\ & + \frac{1}{q_1 - q_2} \int_{-\infty}^t \lambda_2 m_d(\tau) e^{-\lambda_2(\tau-t)} d\tau, \end{aligned} \quad (8)$$

where

$$C_{2d} \equiv \tilde{C}_2 - C_2 = -\frac{1}{q_1 - q_2} \int_{T_0}^{\infty} \lambda_1 m_d(\tau) e^{-\lambda_1(\tau-T_0)} d\tau, \quad \text{and}$$

$$m_d(\tau) \equiv \tilde{m}(\tau) - m(\tau).$$

These equations demonstrate that the divergence of the price level or the capital stock from the path that was previously anticipated is determined in the same way in which the original profiles for the variables were calculated. Since C_{2d} does not depend on the capital stock at T_0 , the innovation in the monetary profile creates divergences in the price and capital profiles which are independent of the current state of the economy.

In light of this conclusion it is clear that others have interpreted their results too narrowly in an important sense. Typically, the economy is assumed to be in a steady-state equilibrium with a constant (usually zero) rate of inflation that is anticipated to occur throughout the indefinite future. Questions regarding the dynamic path of the actual price level or the exchange rate (such as the existence of overshooting of the long-run equilibrium or the need for subsequent adjustment of the economy following an increase in the money supply) depend crucially on the assumption that

the economy is in long-run equilibrium. A more general statement is that against any money supply profile $m(\tau)$ as background, an unanticipated innovation in the future money supply process causes a divergence in the dynamic path of the economy and that divergence has the dynamic properties (such as overshooting) others have ascribed to the path that begins from the steady state.

From (7) and the definition of C_{2d} it is clear that the jump in the price level at the time of the announcement is

$$p_d(T_0) = \int_{T_0}^{\infty} \lambda_1 m_d(\tau) e^{-\lambda_1(\tau - T_0)} d\tau, \tag{9}$$

which depends on the appropriately discounted innovations in the money supply where the discount rate is λ_1 . Thus, any two money supply profiles with the same discounted innovations cause the same jump in the price level.

4. Delays in implementation

The previous section has demonstrated how any innovation in the future money supply profile affects the divergence of the economy from its previous equilibrium path. The generality of that argument prevented precise determination of the effects except for the instantaneous jump in the price level. While this section retains the general expression for innovations in monetary policy, it analyzes an important particular method of implementation. Specifically, here we consider the special case in which the beginning of the innovation or the implementation of the new policy occurs at some time in the future. Specific examples of such policies have been considered frequently in the literature since they highlight the importance of the perfect foresight assumption.¹⁰ We will first discuss the general case of delayed implementation, and later we will examine specific policy experiments.

Assume that the first innovation in the money supply profile occurs at time T_1 , implying that $m_d(\tau) = 0$ for $T_0 \leq T < T_1$. In this case the solutions in (7) and (8) for the price level and the capital stock between the announcement and implementation of the policy are the following:

$$p_d(t) = I(T_1) e^{-\lambda_1(t - T_0)} \left\{ \frac{q_1 e^{\lambda_1(t - T_0)} - q_2 e^{\lambda_2(t - T_0)}}{q_1 - q_2} \right\}, \tag{10}$$

¹⁰Whether the discovery by economic agents that policy is to be changed in the future is via an announcement or by some other means is irrelevant. Fischer (1979) refers to these policy experiments as partially anticipated. Hall (1971), Sargent and Wallace (1973), Brock (1974), Blanchard (1981), Abel and Blanchard (1979), Wilson (1979), and Boyer and Hodrick (1982) all examine perfect foresight models with this type of experiment to highlight the effects produced by the expectations assumption.

and

$$k_d(t) = I(T_1)e^{-\lambda_1(T_1 - T_0)} \left\{ \frac{e^{\lambda_1(T_1 - T_0)} - e^{\lambda_2(t - T_0)}}{q_1 - q_2} \right\} \quad (11)$$

for $T_0 \leq t \leq T_1$. The term $I(T_1)$ represents the value of the innovations in the money supply profile discounted back to T_1 , that is

$$I(T_1) = \int_{T_1}^{\infty} \lambda_1 m_d(\tau) e^{-\lambda_1(\tau - T_1)} d\tau. \quad (12)$$

Clearly, the jump in the price level at T_0 is $p_d(T_0) = I(T_1)e^{-\lambda_1(T_1 - T_0)}$.

Expressions (10) and (11) show that an unanticipated innovation in monetary policy that has a delayed implementation causes time paths for the divergences of the price level and the capital stock during the delay which are of a simple form. Namely, the time paths are all scalar multiples of an exponential function. For now, note that this function depends only on the interval of time following the announcement of a new policy and on the structural parameters of the model. The scalar coefficient depends on the total time between announcement and implementation in the sense that it discounts the value of the money supply innovations which begin at T_1 back to the time of announcement at T_0 .

It is clear from the form of this function that during the delay in implementation, all policies produce time paths for the divergences of the price level and capital stock from their previous equilibrium time paths that are scalar multiples of each other. Thus, any policies that have discounted values which are the same produce an identical jump in the price level and identical time paths until the implementation of the first policy.

The exponential function in (10) and (11) has a straight-forward interpretation. The movement of the economy during the delay in implementation, as described by the divergences of the price level and capital stock from their former equilibrium time paths, is given by the solution to (3) and (4) with zero values for the forcing function. The solution to this problem requires values for two arbitrary constants, D_1 and D_2 .¹¹ These coefficients are found from the initial condition on the capital stock, $k_d(T_0) = 0$, and from the terminal condition that at time T_1 the economy must satisfy the equation¹²

¹¹The solution for $p_d(t)$ and $k_d(t)$ for $T_0 \leq t \leq T_1$ is

$$p_d(t) = D_1 q_1 e^{\lambda_1(t - T_0)} + D_2 q_2 e^{\lambda_2(t - T_0)}, \quad k_d(t) = D_1 e^{\lambda_1(t - T_0)} + D_2 e^{\lambda_2(t - T_0)},$$

where D_1 and D_2 are determined from initial and terminal conditions.

¹²Since the transition paths are scalar multiples, there is only one parameter in (10) and (11) that is not structural in the sense of Hurwicz. The structural parameters in (10) and (11) are λ_1 , λ_2 , q_1 , and q_2 , since they are invariant to changes in policy.

$$p_d(T_1) = q_2 k_d(T_1) + I(T_1) \quad (13)$$

which can be derived by multiplying (8) by q_2 , subtracting it from (7), and evaluating the resulting equation at $t = T_1$.

The results in this section indicate that the movement of the economy during the transition period between announcement and implementation of a new policy is not invariant to the type of policy that is being implemented and is consequently not structural. Nevertheless, the simplicity of the path and the fact that all paths are scalar multiples of each other suggests that these periods may be ones in which we can easily learn about the structure of the economy.

5. A diagrammatic presentation

The previous section has considered the characteristics of the movement of the economy in response to general innovations in future monetary policies. The purpose of this section is to introduce a diagram, presented in fig. 1, that nicely complements the algebraic presentation. For both the closed and the open economy versions of the model, the logarithm of real balances (the log of nominal balances minus either the log of the price level or the exchange rate) is on the vertical axis while the horizontal axis is the logarithm of either the capital stock or ownership of net foreign assets.

Over any period during which the money supply grows at a constant rate, eqs. (3) and (4) can be solved for the long-run values of real balances and the capital stock, and a diagram of the economy's movement will be similar to that in fig. 1. In that figure the line LL is the locus of points for which the money market is in equilibrium given that the rate of inflation is equal to the value of the constant rate of monetary growth. It is positively sloped since an increase in capital increases the demand for money and real balances must increase to keep the money market in equilibrium. The locus of points for which the capital stock or net bond holdings is constant is labelled GG . It is negatively sloped to reflect the fact that increases in wealth reduce the rate at which capital is produced or acquired from abroad.¹³ In this section for convenience only, we consider an economy that is initially at steady-state equilibrium with a constant nominal money stock, m_0 . That equilibrium is at the intersection of LL and GG , denoted in the figure by Q_0 .¹⁴

The curved arrows in fig. 1 indicate the path of the economy when it is

¹³Brock (1974), Mussa (1976), and Begg (1980) all employ this type of diagram. In Brock and Mussa, $k=0$ is vertical reflecting the absence of the Tobin effect in some models with maximizing behavior.

¹⁴It should be clear from the previous section that the diagram also can be used to discuss the time path of the divergence of the economy from its former equilibrium path.

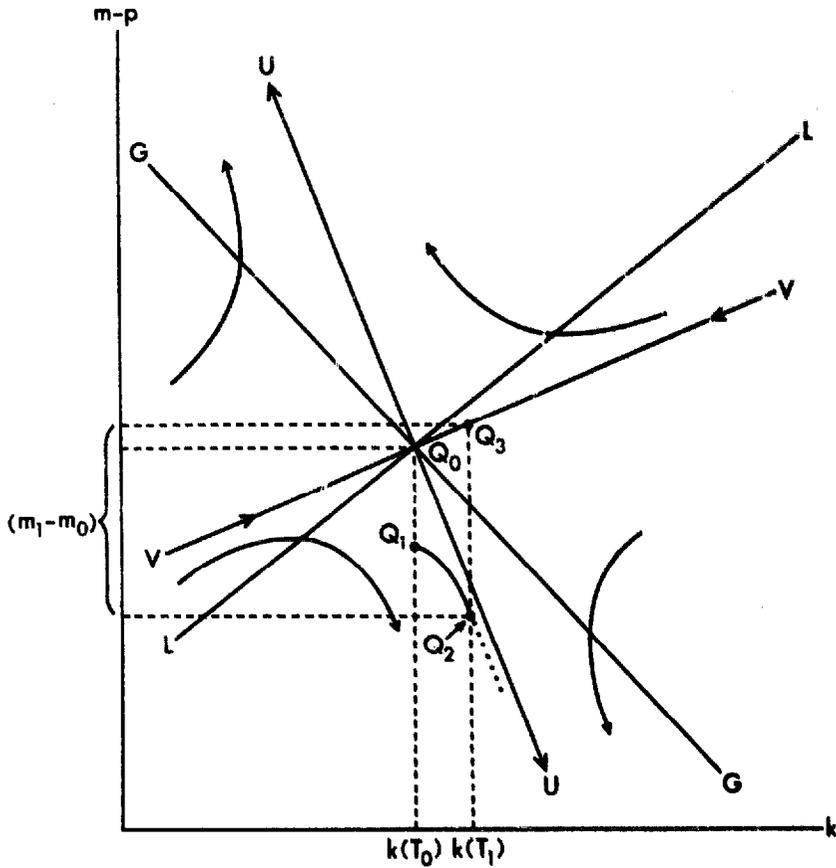


Fig. 1. The steady-state and dynamic forces of adjustment.

away from the steady state. To the left of LL , real balances are increasing since a decrease in the capital stock implies that a lower rate of inflation is necessary to keep the money market in equilibrium. Conversely, real balances are declining to the right of LL . To the left of GG , the stock of assets is increasing since w is lower than its target level.

The lines VV and UU in fig. 1 are the characteristic vectors for Q_0 . These vectors are the stable and unstable steady-state paths of the economy. If the economy is ever on either locus, it remains on it so long as the monetary growth rate does not change, moving at a speed that is proportional to the distance from Q_0 . Along VV , the vector corresponding to the negative root of the system (with slope equal to $-q_2$) the economy moves toward Q_0 . Along UU , the vector corresponding to the positive root of the system (with slope equal to $-q_1$), the economy moves away from Q_0 .

It was argued earlier that the price level or the exchange rate cannot be anticipated to move discontinuously, but it can jump with unanticipated changes in policy. Since the capital stock moves continuously at all points in time, any jumps in the diagram must be vertical, either up or down.

Consider the effect of an announcement of a previously unanticipated step increase in the money supply. This policy is neutral in the long run, and if it is implemented simultaneously with the announcement, there will be an instantaneous equiproportional jump in the price level or the exchange rate leaving the economy at Q_0 . Nevertheless, if there is a delay between the announcement and the implementation of the policy, there will be real effects on the economy since the price level cannot jump at T_1 , the time of implementation.

An anticipated step increase in the money supply causes the economy to jump from Q_0 to Q_1 . Thus, a future increase in the money supply causes an instantaneous upward jump in the price level. During the interim between announcement and implementation, the economy follows the lines of force in fig. 1 moving from Q_1 to Q_2 , a path that corresponds to (10) and (11) with the substitution $I(T_1) = m_1 - m_0$. This is a period in which the rate of inflation increases and people substitute out of real balances and into capital or foreign assets. At T_1 the step increase in the nominal money supply increases real balances causing the economy to jump to Q_3 on the stable path VV . After that moment the economy moves along VV , the stable branch, back to Q_0 . In the process there is a positive but continuously decreasing rate of inflation, and a corresponding lowering of the capital stock.

Dornbusch (1976), Wilson (1979), and Gray and Turnovsky (1979) have examined a somewhat different model of exchange rate dynamics. Their system has a goods market in which prices are 'sticky', moving in a continuous fashion in response to excess demand. In contrast, the exchange rate can jump at a point in time. In that model a step increase in the money supply, whether implemented simultaneously with the announcement or delayed, always causes the exchange rate to 'overshoot' its long-run value with the maximum divergence occurring at the time of implementation. It is interesting to note that in this full employment model no such overshooting occurs since the exchange rate continues to depreciate after the implementation.¹⁵

If the delay in implementation is longer, the initial jump will be smaller and the economy will accumulate more capital during the interim. Alternatively, if the increase in the money supply is larger and the delay is longer, the economy could follow the path $Q_1 Q_2$ extended as a dotted line in fig. 1. Notice that during the interim the dynamic path of the economy can never cross UU .¹⁶ This will be exploited in the next section when changes in the steady state are analyzed.

¹⁵This point was made in Flood (1979) who discusses 'overshooting' of the equilibrium path of the exchange rate caused by a change in the rate of growth of the money supply. See section 6 for an elaboration.

¹⁶This fact could have been used by Wilson (1979) in both his diagrammatic and analytical discussion of delayed implementation in the Dornbusch (1976) model.

6. Alternative monetary policies

The previous section demonstrated how a well-known diagram can be used to analyze the effects of changes in monetary policy that are announced at one time and implemented at some future date. This section considers two other policy experiments: a temporary step increase in the money supply, and an increase in the rate of growth of the money supply. Both of these experiments are unanticipated at T_0 when they are announced. Until they are implemented at T_1 , agents believe that the money supply will be constant at m_0 . The analysis of these cases is presented in fig. 2.

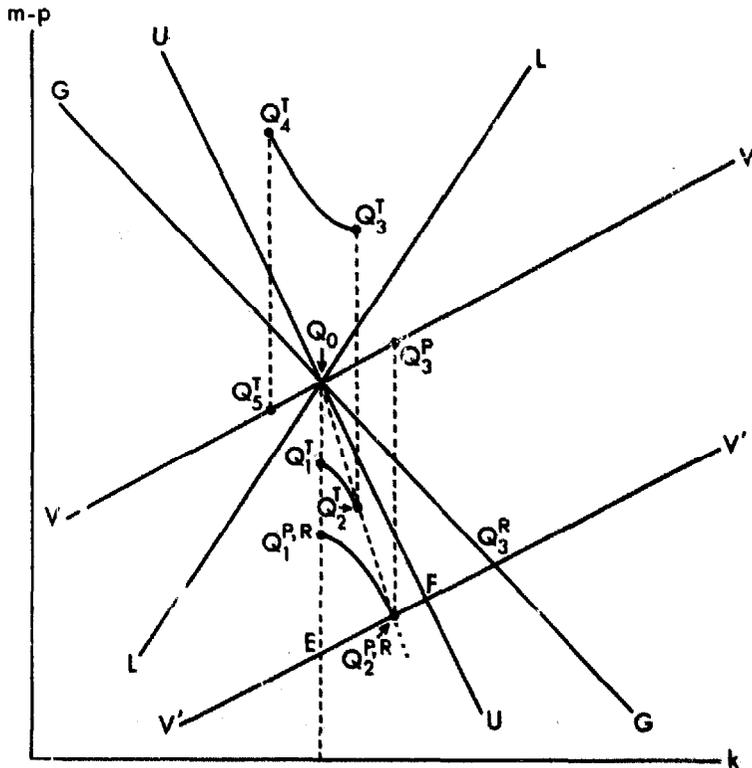


Fig. 2. Dynamic response to alternative money supply policies.

Rather than presenting the analytical time path for each policy, recognize that (10) and (11) describe the economy between announcement and implementation, where $I(T)$ needs to be derived in each case. After the final policy change, at time T_i , the time path of the economy is

$$p(t) = m(t) - \overline{(m-p)} + H \frac{q_2 e^{\lambda_2(t-T_i)}}{q_1 - q_2}, \tag{14}$$

$$k(t) = \bar{k} + H \frac{e^{\lambda_2(t-T_i)}}{q_1 - q_2}, \tag{15}$$

where $\overline{(m-p)} = -\mu\beta_2/(\beta_1\alpha_2 + \beta_2\alpha_1)$ and $\bar{k} = \mu\beta_1/(\beta_1\alpha_2 + \beta_2\alpha_1)$ are the long-run values for real balances and the capital stock that the economy approaches when there is a constant rate of monetary growth equal to μ . The coefficient H is determined by $H/(q_1 - q_2) = k(T_i) - \bar{k}$, where the right-hand side of this expression is the deviation of k at T_i from its long-run value. Similarly, $-Hq_2/(q_1 - q_2)$ is the deviation of real balances at T_i from their long-run value. T_i will equal T_1 for a permanent step increase or an increase in the rate of growth of the money supply and will equal T_2 for a temporary step increase since the decrease in the money supply at that time is a policy change in our nomenclature. Eqs. (14) and (15) demonstrate that the economy converges to its long-run equilibrium along the stable path after the final policy change. Table 1 presents the values for $I(T_1)$ and H for the three policies. Finally, we present below the analytical time path of the economy for the period between T_1 and T_2 in the case of the temporary increase.

Table 1*

Policy	$I(T_1)$	H
Permanent step increase	$(m_1 - m_0)$	$(m_1 - m_0)\{1 - e^{(\lambda_2 - \lambda_1)(T_1 - T_0)}\}$
Temporary step increase	$(m_1 - m_0)(1 - e^{-\lambda_1(T_2 - T_1)})$	$-(m_1 - m_0)\{1 - e^{\lambda_2(T_2 - T_1)}\} \{-e^{(\lambda_2 - \lambda_1)(T_1 - T_0)}\} / (1 - e^{-\lambda_1(T_1 - T_0)})\}$
Increase in rate of growth	μ/λ_1	$\mu \left\{ \frac{1 - e^{(\lambda_2 - \lambda_1)(T_1 - T_0)}}{\lambda_1} \right\}$

* $I(T_1)$ represents the size of a particular policy innovation as described in (12). It is introduced in (14) and (15). The deviation of $k(T_1)$ from \bar{k} is $H/(q_1 - q_2)$.

In order to analyze that case, consider a temporary step increase in the money supply of the same magnitude as the permanent increase of the previous section. Specifically, assume that at T_0 the money supply is increased from m_0 to m_1 , and at T_2 the money supply is returned to m_0 . The impact of the announcement is to cause an immediate jump in the price level equal to $(m_1 - m_0)(1 - e^{-\lambda_1(T_2 - T_1)})e^{-\lambda_1(T_1 - T_0)}$. Notice that this is less than the jump when the increase is permanent. In terms of fig. 2, the economy jumps from Q_0 to Q_1^T , showing that a temporary increase in the money supply produces the same qualitative behavior in the economy between announcement and implementation as does a permanent increase. As noted above, the two paths must be scalar multiples of each other, in this case their ratio is equal to $(1 - e^{-\lambda_1(T_2 - T_1)})$. For convenience, we repeat in this figure the path of a permanent step increase in the money supply, labelling it as $Q_1^{P,R} Q_2^{P,R}$. Notice that at T_1 the position of the economy is along the same ray from the initial equilibrium, consistent with the paths being scalar multiples.

Concentrating attention once again on the case of a temporary increase, note that at time T_1 the increase in the money supply jumps the economy to Q_3^T . From that point a deflation of the price level pushes the economy to Q_4^T ; in the process there is a decumulation of assets. During this period the analytical time path of the economy is

$$p(t) = m_1 - \frac{(m_1 - m_0)}{q_1 - q_2} \{q_1 e^{-\lambda_1(T_2 - t)} - q_2 e^{\lambda_2(t - T_1)}\} \\ \times [1 - e^{(\lambda_2 - \lambda_1)(T_1 - T_0)}(1 - e^{-\lambda_1(T_2 - T_1)})], \quad (16)$$

and

$$k(t) = -\frac{(m_1 - m_0)}{q_1 - q_2} \{e^{-\lambda_1(T_2 - t)} - e^{\lambda_2(t - T_1)} [1 - e^{(\lambda_2 - \lambda_1)(T_1 - T_0)}] \\ \times (1 - e^{-\lambda_1(T_2 - T_1)})\}. \quad (17)$$

At T_2 , when the money supply is returned to its original level, the economy must be on the stable path so as to converge back to its equilibrium at Q_0 . The reduction in the money supply moves the economy from Q_4^T to Q_5^T , and the economy accumulates capital while accumulating real balances through a deflation.¹⁷ The movement of the economy caused by the temporary step increase demonstrates the important point that the existence of the Tobin effect does not imply inflation and capital accumulation or depreciation of the exchange rate and a current account surplus must always coincide. Only when monetary growth is anticipated to be constant over the indefinite future can the economy's path be characterized by deflation with accumulation of assets or inflation with decumulation of assets.¹⁸

Consider now an increase in the rate of growth of the money supply to some positive rate μ . Eqs. (16) and (17) demonstrate that the new steady state will be characterized by lower real balances and a larger capital stock. At the time of the announcement the economy jumps to $Q_1^{P,R}$, since we have drawn fig. 2 for the special case $(m_1 - m_0) = \mu/\lambda_1$. For a change in the rate of growth of this magnitude, the time path of the economy between announcement and implementation coincides with the time path produced by a permanent step

¹⁷That the final approach in the case of a temporary increase in the money supply is always from the southwest rather than the northeast confirms the finding in Fischer's model that the reduction in the anticipated rate of inflation is an unambiguously stronger effect than the accumulation of capital prior to the initial implementation of the policy.

¹⁸Kouri (1976) examines his model under alternative assumptions on expectations formation. He notes that the perfect foresight path is always along the stable branch, *VV*. Unfortunately, his statement that depreciation (appreciation) of the exchange rate and current account deficits (surpluses) always occur together in a perfect foresight model is incorrect as this section demonstrates.

increase of $m_1 - m_0$. Since there are no other policy changes after T_1 , the economy arrives at $Q_2^{P,R}$ on the new stable branch precisely at that time. After T_1 , the economy converges to its new long-run equilibrium at Q_3^R .

The announcement of a change in the rate of growth of the money supply causes an initial jump in the price level and an inflation between announcement and implementation. After the increase in the rate of growth the economy accumulates real balances since inflation is slower than μ . In this sense the price level or the exchange rate can be said to 'overshoot' its long-run equilibrium.¹⁹ It is interesting to note that the longer the delay between announcement and implementation the closer is the economy to Q_3^R , but there is always a non-trivial adjustment, FQ_3^R in fig. 2, which occurs subsequent to the implementation. If the implementation occurs at T_0 , the economy jumps to E and the price level overshoots by the largest amount.

7. Conclusions

This paper has investigated non-neutralities of anticipated monetary policy in a rational expectations environment. The major finding of the paper is the structural nature of the time path of the economy between announcement and implementation of policy. We demonstrated that the transition path between the time agents are aware of the authorities' intention to alter policy instruments and the time that implementation occurs depends only upon the length of the delay and the size of the policy change. Size is described by the integral of future changes in policy discounted to the time of implementation.

The analysis demonstrated that it is useful to examine the effects of an unforeseen policy change in terms of the divergence of policy instruments from their previously anticipated values. This divergence causes all endogenous variables to deviate from their former anticipated time paths, and in a linear model, these deviations can be analyzed independently from the original anticipated time path. When expressed as deviations from initial equilibrium paths, it is clear that the transition paths described above are all scalar multiples of each other.

Another important finding of the analysis was that along the transition path, real balances and the capital stock need not move in the same direction as they do after the implementation. Hence, the strong prediction regarding the co-movements of real balances and capital of models which have not considered delayed implementation must be modified.

Sargent (1976) first noted the observational equivalence of the new classical macroeconomic theories of the business cycle and the older neo-Keynesian

¹⁹Clearly, if real balances equalled their long-run value at T_1 , the price level would grow at the same rate as the money supply. Since real balances are below their long-run level, the price level is 'too high' and the subsequent adjustment requires a rate of inflation less than μ . See the discussions in Flood (1979) and Boyer and Hodrick (1982) for further analysis of this issue.

macroeconomic models when policies are constant. Neftci and Sargent (1978) examined a change in monetary policy because the implications of the models are different at such policy 'breaks'. An implication of this paper is that characterizing how policy is formed and what changes in the money supply are known to occur in the future is an important exercise since we must control for the non-neutralities of anticipated monetary policy in testing the new classical theories.

An additional implication of the model reinforces Sargent's insight that it is unforeseen changes in policy that provide the most fruitful ground for testing the implications of rational expectations models. Our reasoning suggests that the transition path for delayed policy implementation may provide an excellent testbed for discovering the importance of the Mundell-Tobin effects and for examining the validity of the rational expectations model.

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